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# STRESS-STRAIN MODEL FOR FRP-CONFINED RECTANGULAR RC SECTIONS VIA AN INCREMENTAL PROCEDURE

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**ABSTRACT**
One of the best performances of FRPs is known to be their application in confining concrete. The case of FRP-confined concrete in circular columns is well studied and the performance of already existing models is nowadays considered as satisfactory. On the other hand, the case of FRP-confinement of rectangular sections is a more complex problem, whose physical behavior is still far from being fully understood. The aim of this study is to try and simplify the problem and to propose an iteration procedure based on the outcome of 3D FEM analysis run by the authors. An interesting outcome is that the so-called *arching effect* is never observed: indeed, the unconfined regions are partially confined and provide a certain contribution to the overall strength of the rectangular sections. Based on a system of generalized springs – and also on well-known stress-strain laws and a failure criterion – a simplified mechanical model that gives the stress-strain behavior of a rectangular RC section under concentric load is proposed. This can easily be understood and implemented by designers. Its predictions satisfactorily correlate to experimental results, taking into account all parameters, such as corner rounding radius, aspect ratio of the section, stiffness of the FRP and concrete strength.

**KEYWORDS**

FRP Confinement, rectangular section, model, stress-strain behavior.

**INTRODUCTION**

The behavior of fiber reinforced polymer (FRP) - confined concrete in circular columns has been widely studied, and now the efforts of many researchers are directed towards the comprehension of the case of FRP-confined rectangular columns. Here, concrete is non-uniformly confined and the confinement effectiveness is remarkably reduced. In the literature, past approaches have concentrated on dividing the rectangular section in a confined and an unconfined area, based on the idea of possible arching effect between corners. Then, the confined zone is considered to be in a state of uniform biaxial confinement, like in the circular cross section, thus allowing the use of formulas derived for circular columns, while the unconfined zone is considered unaffected. This different behavior is usually accounted for through so-called effectiveness coefficients, mainly based on geometrical considerations on the relative size of the two different zones. In this study, an alternative approach is attempted, which recognizes, through an iteration procedure based on the outcome of 3D FEM analysis, that the arching effect doesn't really exist. The unconfined regions are indeed partially confined and contribute, though to a lesser extent with respect to the core, to the total strength of the rectangular sections.

**NUMERICAL ANALYSIS (FEM)**

In a paper of Campione and Miraglia (2003) the picture below (Figure 1, left) is reported. The figure is related to the cross-section of a short square column and shows the effective concrete core after FRP failure at the corner. Uneven damage can be observed throughout the section and two different regions can be identified having different confinement stress state. The peripheral region is usually identified as an unconfined one, while it will be shown later that this is actually a partially confined region. The path of the confining stresses in the peripheral region was studied with a 3D FEM in SAP 2000 in the linear range (Figure 1, right).

Figure 1. Cross-section of a square column: left, test (from Campione and Miraglia 2003); right, FEM model.

From the FEM results, 3D plots were drawn of the stress fields (Figure 2) in a quarter section (100x100 mm). Stresses near the rounded corner at (0,0) coordinates are not shown, since they represent local stress states. Stresses parallel to the diagonal (SD1 and SD2) and stresses parallel to the sides (SO1 and SO2) are presented separately, so to better recognize the direction of the confining stresses. It can be noticed that, though of lower intensity, stresses parallel to the sides do exist that confine the concrete in the peripheral regions.

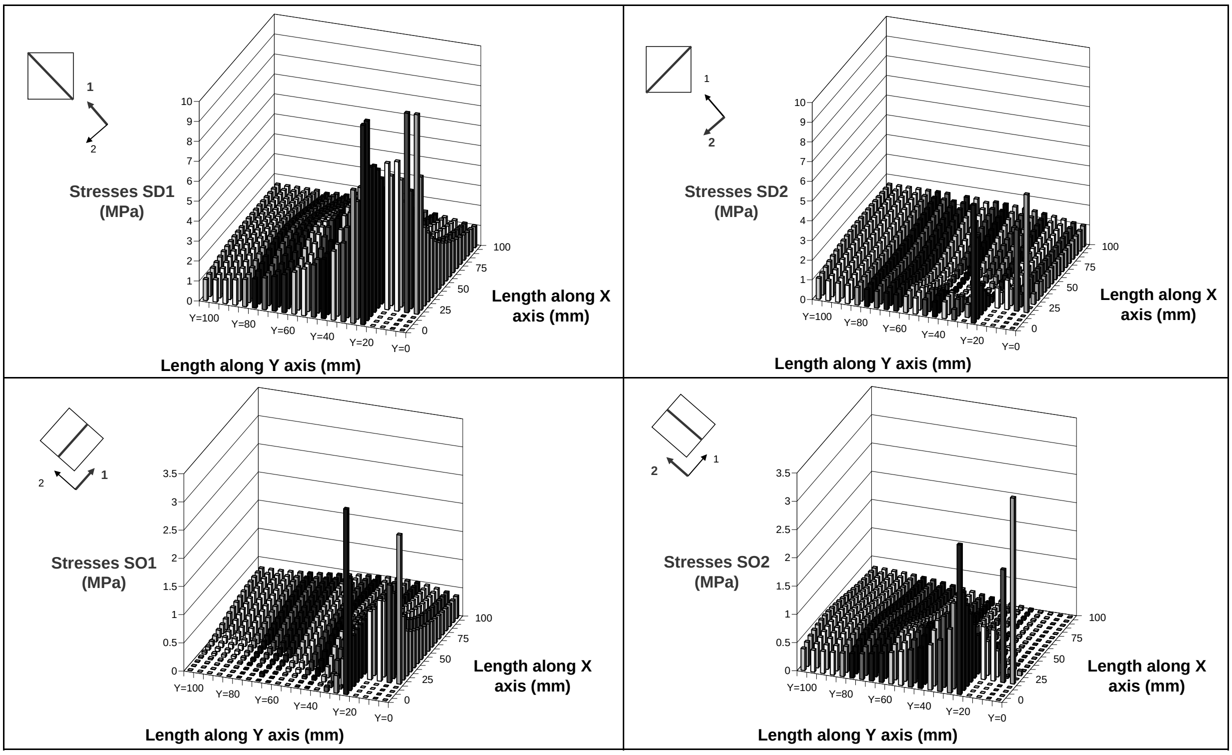


Figure 2. Confining stresses in a quarter of a square section 200x200 mm:
along diagonal (top) and orthogonal (bottom) direction. The round corner stresses are not included.

Observing the plots, the following important remarks can be made:

- No unconfined concrete regions are observed, as assumed in many models. The peripheral regions near the sides are confined from stresses coming from the corners and acting parallel to the sides.
- The confining stresses in the peripheral regions have strong directionality, *i.e.*, they mainly act along the sides (uniaxial confinement), while near the section centre they are more uniform (biaxial confinement).

The regions with either uniaxial or biaxial confinement can be easily identified, for example, based on the principal stresses ratio: in this case, the regions where such ratio is lower than 15% are considered as uniaxially confined. It has been observed that the size of the regions is independent of the FRP axial stiffness, as long as its flexural stiffness is negligible (which is always the case). Moreover, the rounded corner radius does not seem to affect the size of the uniaxially confined peripheral region and, for the sake of simplification, its width can be simply taken as 1/8 of the corresponding side (Figure 3, where $h_1$=$h_2$=$h$/8, with $h$=side). The dimensions of the other regions are then easily determined. The next step is to assign them meaningful mechanical properties.

## SIMPLIFIED MECHANICAL MODEL

The FRP-confined RC square section is here modeled by a series of generalized springs (Figure 3) describing the following behavior: compressed concrete expands laterally according to its confinement state, either uniaxial or biaxial; such expansion activates the confining device, which in turn applies the confinement stress.

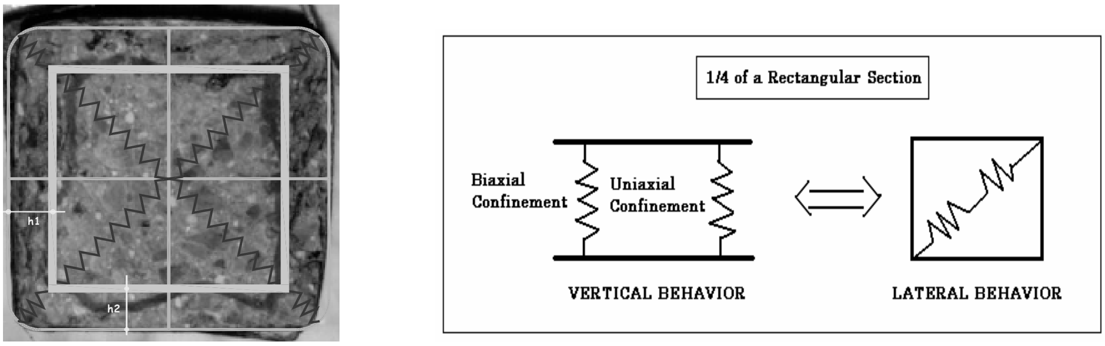


Figure 3. Generalized springs describing the 'vertical-lateral' behavior of biaxially/uniaxially confined regions.

The confining forces are applied at the section corners and act along the diagonal (on passing, they are always at 45° angle, even for rectangular sections). Figure 3 illustrates how the springs model the section's vertical (axial) and lateral behavior. Vertically, the springs undergo the same displacement and their forces are added (parallel system); laterally, the springs undergo the same force and their displacements are added (series system).

The behavior of these generalized springs is described by the model of Pantazopoulou and Mills (1995), relating volumetric strains $\varepsilon_V$ to axial strains $\varepsilon_c$. In order to comply with the generalized springs model and the differently confined regions, the model has been modified into a 'volumetric strain - axial stress' ($\varepsilon_V$ - $\sigma_c$) constitutive law, to obtain the following equation (the linear-parabolic behaviour of 'volumetric strain - axial strain' has been experimentally observed in the 'volumetric strain - axial stress' behaviour, as well):

$$\varepsilon_V = -(1-3\nu)\cdot 10^{-4}\cdot\sigma_c \qquad \sigma_c \le \alpha f_{co} \tag{1a}$$

$$\varepsilon_V = -(1-3\nu)\cdot 10^{-4}\cdot b\cdot f_{co}\left[\frac{\sigma_c}{b f_{co}} - \left(\frac{\sigma_c - \alpha f_{co}}{b f_{co} - \alpha f_{co}}\right)^c\right] \qquad \sigma_c > \alpha f_{co} \tag{1b}$$

The above equations describe the following behavior: initially, volume change is of compaction and is almost linear up to the critical stress of $\alpha f_{co}$ ($f_{co}$=unconfined concrete strength, usually with $\alpha$=0.7). Note that for this axial stress level (where the initial Young's modulus of concrete is usually determined, as well), the Poisson's ratio $\nu$ remains in the range of 0.15-0.25. Beyond this state, there is volumetric expansion, called (near- or at-peak-strength) dilatancy. A point can be found where the volumetric strain is equal to zero. This point is considered to coincide with the attainment of the ultimate strength of the uniaxially confined region ($b$=1.2). After this region degrades, the expansion rate increases more than the compression rate (second order parabola, $c$=2) due to less effective confinement, so that the expansion becomes unstable during the crushing phase beyond the peak strength.

From the volumetric strain, both area $\varepsilon_A$ and side $\varepsilon_{a,b}$ strains can be easily calculated as follows (compressive axial strains are taken as negative, and $a$ and $b$ are the sides length, related by: $b/a$ = tan$\theta$, with $\theta$ the diagonal angle):

$$\varepsilon_A = \varepsilon_V - \varepsilon_c \tag{2}$$

$$\varepsilon_A = \frac{\Delta A}{A} = \frac{(a+\Delta a)\cdot(b+\Delta b) - a\cdot b}{a\cdot b} = \frac{(a+\varepsilon_a\cdot a)\cdot(b+\varepsilon_b\cdot b) - a\cdot b}{a\cdot b} = (1+\varepsilon_a)\cdot(1+\varepsilon_b) - 1 \tag{3}$$

$$\frac{\varepsilon_a}{\varepsilon_b} = \frac{\Delta_a}{a}\frac{b}{\Delta_b} = \frac{\Delta_{diag}\cdot\cos\theta\cdot b}{\Delta_{diag}\cdot\sin\theta\cdot a} = 1 \quad\Rightarrow\quad \varepsilon_{side} = \varepsilon_a = \varepsilon_b \tag{4}$$

$$\varepsilon_A = (1+\varepsilon_{side})^2 - 1 \quad\Rightarrow\quad \varepsilon_{side} = \sqrt{\varepsilon_A + 1} - 1 \tag{5}$$

By the usual no-slip assumption between FRP jacket and concrete, sides strains $\varepsilon_{side}$ and FRP jacket strain can be equated. Thus, the diagonal force applied by the FRP jacket from the corners to the springs, both diagonal and parallel to the sides, can be found as:

$$F_{diag} = \sqrt{2} \cdot E_j \cdot \varepsilon_{side} \cdot t_j \cdot k_e \tag{6}$$

$$k_e = \frac{R}{a} \cdot \frac{1}{2} \cdot \left(1 + \frac{a}{b}\right) \tag{7}$$

where $E_j$ and $t_j$ are the jacket Young's modulus and thickness, respectively; $k_e$ is an efficiency factor (Karam and Tabbara 2005), which considers that confinement effectiveness increases as the corner radius $R$ increases, while it decreases as the aspect ratio ($a/b$, where $a$ and $b$ are the section sides) of the section changes. Having determined the diagonal force applied from the corners to the lateral springs in series, the lateral pressures (assumed as uniform) for each region can be determined (Figure 4). Two regions are identified: one with a biaxial stress state (uniaxial confinement) and one with a triaxial stress state (biaxial confinement). Note that there are no unconfined regions.

Based on the lateral force calculated in (6), the corresponding vertical stress can be easily determined by the use of a biaxial or a triaxial stress-strain model corresponding to the confinement stress state of each region. For the biaxial stress-state region, the model for concrete under biaxial stress-state of Liu et al. (1972) is used. For the triaxial stress-state region, the stress-strain model by Popovics (1973) is used. In this latter, in order to account for different confining pressures in the two directions (they are equal only in square sections), the failure criterion by Ottosen (1977) is applied.

Finally, from the areas of the different regions, the total averaged axial stress of the section is computed.

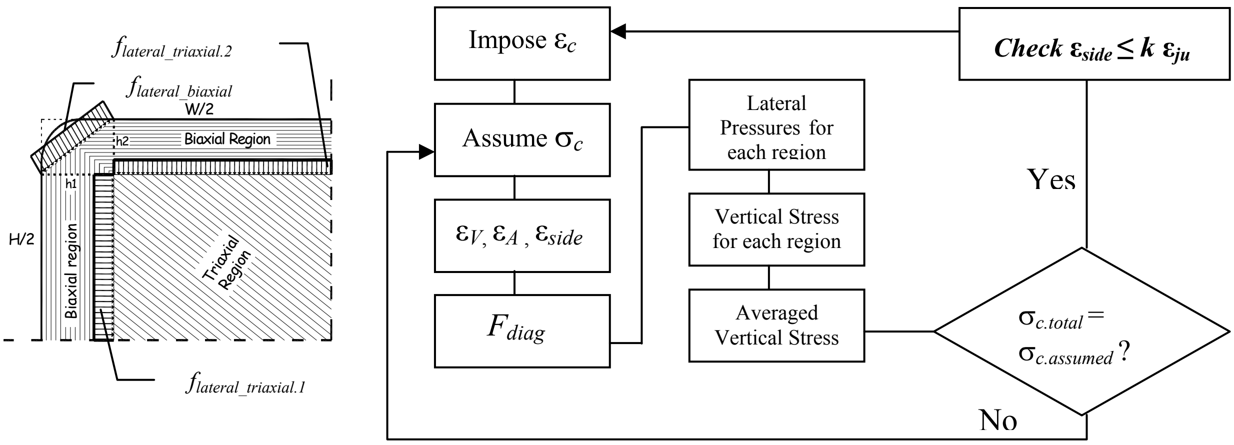


Figure 4. Confining stresses in the regions – Iteration procedure.

The above procedure, which is iterative, is shown in Figure 4, where an assumed value of axial stress $\sigma_c$ corresponding to an imposed axial strain $\varepsilon_c$ is brought to convergence. After convergence is reached, the resulting sides strain through the iteration procedure are compared to the rupture strain of the jacket. It has been observed from experimental results (Lam and Teng 2003) that the average failure strains of the FRP wraps are of the order of 50-80% of the failure strain of the tensile coupons made from the same material and tested before the application of the material. The value of the factor $k$ in Figure 4 depends on the type of FRP used.

## CORRELATION WITH EXPERIMENTAL RESULTS

Two series of experimental results have been used that contain all the parameters needed for validating the model. The first is a square short column confined by CFRP (Wang and Wu 2007) and the second are square and rectangular short columns (Chaallal et al. 2000). The correlations shown in Figure 5 are satisfactory for as regards the axial behavior, while they are less accurate in predicting the lateral behavior.

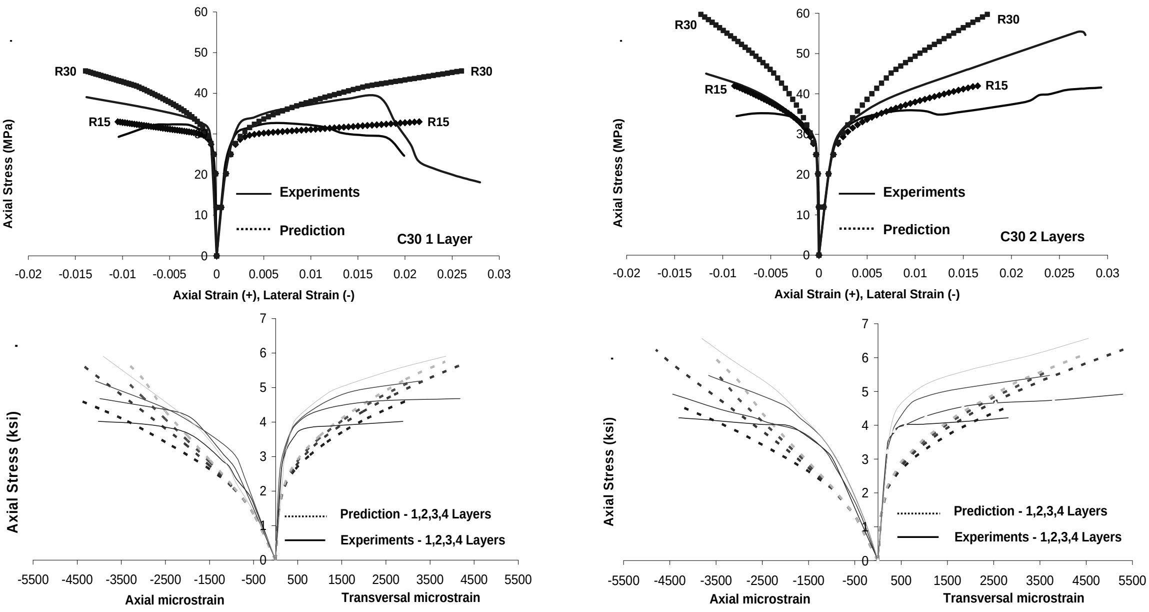


Figure 5. Correlation with two groups of experimental studies. Top: Square specimens ($f_c$=30 MPa). Bottom: Rectangular Specimens (a/b=0.50 (left), a/b=0.65 (right), $f_c$=20.7 MPa).

## CONCLUSIONS

An iterative approach has been proposed to model the stress-strain response, both vertical and lateral, of axially loaded FRP-confined square and rectangular reinforced concrete sections.

A meaningful conclusion is that in confined square/rectangular sections, no unconfined concrete regions exist, as instead usually assumed in many design models (where usually an arching effect is assumed in the shape of arc segments). It has been shown that the regions along the sides are indeed partially confined by the stresses coming from the corners and acting parallel to the sides and, as such, they contribute to the global strength increase, though to a lesser extent than the core regions. The two different regions, peripheral and core, have different confining stress-states: the former are uniaxially confined (biaxial stress state) and the latter are biaxially confined (triaxial stress state). Their joint behavior has been modeled through a system of generalized springs, whose vertical forces, based on the relevant constitutive law, are added.

The lateral behavior develops along the diagonals of the section and can be represented by a system of springs in series. It has been proven that the sides' strains of rectangular sections are equal, regardless of the aspect ratio. The reacting force of the confining device applied from the corners can be shared among the regions based on the path of the confining forces and the geometry of the regions. The resulting confining stressed in each region determine the corresponding behavior.

Correlation with experimental results turns out to be satisfactory, especially in terms of ultimate strength, which is the key element for the designer, while the lateral behavior needs still to be studied. This is usually a weak aspect of any model, also depending on the extreme difficulty of measuring lateral strains in experimental tests. Further comparisons with more reliable experimental results is therefore needed.

## ACKNOWLEDGMENTS

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